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On the Inverse Gaussian Record Values

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Abstract

Record values have a pivotal role in real life applications. A lot of literature is available on upper and lower record values from various distributions except inverse Gaussian distribution (IGD). In this paper we focus on upper record values from IGD including its pdf and some of its basic properties. It has been proved that upper record values from normal distribution is the limiting case of upper record values from IGD when the shape parameter tends to infinity and the scale parameter equals to one. A recurrence relation in the moments of upper record values from IGD has been developed. A simulation study is conducted to find mean, variance, median, mode and quartiles for upper record values from the IGD. For this purpose, 50 upper record values of IGD has been taken, in 50 random samples of 100 size each drawn from IGD and show that for large values of shape parameter the distribution of upper record values from IGD is symmetrical.

Keywords: IGD, UR-IGD, Survival function, quartiles, recurrence relation, symmetrical.

Introduction

Record is a piece of evidence about past, any document or other source of information recorded in written form by any electronic process or by any other form. For example a project has a number of records like policy and planning documentation, reports, background research material, project estimates and costing, budget information, requirements etc. All these records may be in the form of papers, letters, spreadsheets etc. and they may be saved in a number of files or on systems. If they are used or resulted from the decision making process and activities for the progress of a project, then they are records. Records must be accurate, authentic and useable, and have integrity. Record is more than its informational index. All these characteristics make differences between records and other forms of recorded information.

Any observation is said to be a record value if its value is greater than or less than all the preceding observations. Record values include minutes, reports, memos, student's grade sheets, fliers, news clipping, agendas, purchase order, weights of objects etc. There are various records that might be interested to people such as, the lowest and highest temperatures in a locality for last 10 years, very tall building, oldest building, largest tower, highest water fall, largest stadium, any disaster in a region, richest person of the world etc.

Consider $X_1, X_2, \ldots$ are the infinite sequence of independent and identically distributed random variables with cumulative distribution function $F(X)$ and pdf $f(X)$. Suppose $Y_1=X_1$ and $Y_n = \max(\min)\{X_1, X_2, \ldots\}, n \geq 1.$ We say that X_j, j>1 is called an upper Ilower) recrd value of the sequence $\{X_1, X_2, \ldots.\}$ if $Y_j > (<) Y_{j-1}, j > 1.$ Lower record values can also be defined in the same way. We can transform the upper record value to lower record value by replacing the original sequence of $\{X_j\}$ by $\{-X_j, j \geq 1\}$ or (if $P(X_i > 0) = 1$ for all i) by $\{1/X_i, i \geq 1\}$.

We consider $X_{U(n)}$ and $X_{L(n)}$ are the nth upper and lower record values respectively. The distribution of record values as,

$$S(x) = P(X > x) = 1 - F(x) = \bar{F}(x),$$

where $S(x)$ is the survival function and

$$R(x) = -\ln \bar{F}(x)$$

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$$=-\ln\left[1-F(x)\right],\ 0<F(x)<1.$$

Let $F_n(x)$ be the distribution function of $X_{U(n)}, n\geq 1$, then

$$F_n(x)=P\left(X_{U(n)}\leq x\right)=\int_{-\infty}^{x}\frac{\left(R(u)\right)^{n-1}}{\Gamma n}dF(u), \qquad -\infty<x<\infty$$
.

Therefore pdf $f_n(x)$ of nth upper record value $X_{U(n)}$, is

$$f_n(x)=\frac{\left(R(x)\right)^{n-1}}{\Gamma n}f(x), \qquad -\infty<x<\infty \tag{1}$$

Chandler (1952) introduces the statistical study of record values. He shows that the distribution of number of record values is infinite from any distribution. If weighing of some objects and we have a sequence of observed weights as 9.5, 4.0, 2.9, 8.5, 6.1, 7.3, 10.2 etc. the first upper record value is 2.9, second upper record value is 4.0 and third upper record value is 6.1 and so on. Similarly the lower record values can be defined. Record values can be from order statistics. Both are random variables and arranged in ascending or descending order of magnitude. Record values and associated inference have useful applications in many real life data e.g. weather, sports, economics, life testing studies and stock exchange. The record values are used to construct the mathematical model of records, then fit these models to real data and used for forecasting as, earth quake, floods or any disaster in the locality. Records are memorable in their times and also useful for the progress of science and technology that would be beneficial for mankind.

Ahsanullah (1984) discussed the linear prediction of record values for the two parameter exponential distribution. Ahsanullah (1986) discussed distributional properties of the record values from a rectangular distribution. He obtained BLUE of the parameters of distribution. Ahsanullah (1987) gave various properties of record values from exponential distribution and some characterizations. Ahsanullah and Houchens (1989) discussed distributional properties and estimators of record values from Pareto distribution. Ahsanullah (1992) considered various distributional properties of record values of independent and identically distributed continuous random variables. Balakrishnan and Chan (1993) discussed the record values from the Rayleigh and Weibull distribution and associated inference. Balakrishnan et al. (1995) proposed the logistic record values and associated inference. Ahsanullah and Bhoj (1996) discussed properties of record values from Type-I extreme value distribution and a test statistic is proposed based on record values. Balakrishnan and Chan (1998) proposed the normal records and associated inference. They computed the numeric value of means, variances and covariance of normal record values and determined the BLUEs of the parameters.

Raqab (2000) established the general relations for the expectations of functions of record values that may be used to find the recurrence relations for the moment of record values. Sultan et al. (2008) introduced the estimation and prediction from the gamma distribution based on record values. They computed the numeric moments of these using these moments they found BLUEs for the parameter of gamma distribution. They also discussed the interval prediction for the future records. Khan and Zia (2009) gave some recurrence relations satisfied by single and product moments of upper record values from Gompertz distribution and also a characterization is presented. Ahsanullah (2010) derived the r-th concomitants and joint distribution of r-th and s-th concomitants of record values from the bivariate pseudo-Weibull distribution. He derived recurrence relation for the single moments. Sultan (2010) presented the estimation methods of record values from the inverse Weibull (IW) lifetime model. He also derived the BLUEs of the IWD. He obtained the relative efficiency to compare the different estimates. Shakil and Ahsanullah (2011) investigated the distribution of record values of ratio of two independently distributed Rayleigh random variables. They derived moments, hazard function, and entropy for the ratio of two iid Rayleigh random variables. Teimouri and Gupta (2012) discussed the Weibull record values and associated inference. Shakila and Ahmad (2014) developed record values from the size-biased Pareto distributions and derived its various properties including a characterization. Shakila, Ahmed and Ahmad (2014) established some distribution properties of record values from the two-sided power distribution.

Schrodinger (1915) and Smoluchowsky (1915) give the first passage time distribution of Brownian motion with positive drift. Tweedie (1945) suggested the name inverse Gaussian distribution for the first passage time distribution after noticing that there is inverse relationship between the cumulate generating function of the first passage time distribution and of the normal (Gaussian) distribution. Afterward Wald (1947) derived the

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limiting forms of the distribution and due to this derivation it is said to be standard Wald distribution with one parameter. The IGD had extensive uses in reliability. Meanwhile the first passage time distribution of Brownian motion had an inverse Gaussian distribution.

For the last three decades, the inverse Gaussian distribution had incredible courtesy in describing and analyzing right-skewed data. The distribution was established valuable applications in a wide range of fields, such as biology, economics, remedy, reliability and life testing. The IG distribution had extensive use in describing length and disaster occurrences in the natural and social sciences such as duration of strikes, sanatorium stays, employee service times, utensils lives, blaring strength and Tracer reduction curves. Inverse Gaussian distribution had applications in area of applied study as cardiology, hydrology, demography, linguistics,civilian engineering, industrial engineering, marketing and internet traffic. The inverse Gaussian distribution also used to define the time for a pool or dam to empty, to model the time to failure of a device, the consumption time for a consumable product and the time to death of scarce species. The time consumed by vaccinated labeled substances, called tracer, in a biotic system.

The two parameter family of inverse Gaussian distribution by Tweedie (1957), has the probability density function as follows

$$f(x;\mu,\lambda)=\sqrt{\frac{\lambda}{2\pi x^3}}\exp\left\{-\frac{\lambda}{2\mu^2 x}(x-\mu)^2\right\},\qquad x>0,\lambda>0,\mu>0 \tag{2}$$

where μ is the scale parameter and λ/μ is shape parameter. The density function is unimodal, leptokurtic and positively skewed and seen to be a member of exponential family. The inverse Gaussian distribution has cdf as

$$F(X)=\Pr(X\le x)=\Phi\left[-\sqrt{\frac{\lambda}{x}}\left(1-\frac{x}{\mu}\right)\right]+\exp\left(\frac{2\lambda}{\mu}\right)\Phi\left[-\sqrt{\frac{\lambda}{x}}\left(1+\frac{x}{\mu}\right)\right] \tag{3}$$

where $\Phi(z)$ is the cdf of standard normal distribution function.

The IG distribution shares elegant properties with normal, lognormal, gamma, chi-square, Weibull and other skewed distribution. For some special cases the IGD approaches to normal, chi-square and lognormal distributions. Inverse Gaussian distribution is chosen over log normal due to the strong appearance of the exact sampling theory of IGD. The main appeal of using the inverse Gaussian distribution is based on these facts: first a charm to the fundamental physical properties of the procedure being modeled, secondly, the concept of failure rate and their asymptotic performance, thirdly, controllability of the sampling distribution of the IGD.

Materials and methods

Upper record values from the inverse Gaussian distribution

Let $X_{U(1)},X_{U(2)},\cdots,X_{U(n)}$ denote the upper record values arising from the iid inverse Gaussian variables if $X\sim IG(\mu,\lambda)$, and then using equation (1), the probability density function of the nth upper record value $X_{U(n)}$ from inverse Gaussian distribution (UR-IGD) is given by

$$f_n(x)=\frac{\sqrt{\lambda/2\pi}}{\Gamma n}x^{-\frac{3}{2}}\exp\left(-\lambda(x-\mu)^2/2\mu^2 x\right)$$
$$\left[-\ln\left(\Phi_1(x)-\exp(2\lambda/\mu)\Phi_2(x)\right)\right]^{n-1},\; x>0 \tag{4}$$

where,

$$\Phi_1(x)=\Phi\left(\sqrt{\frac{\lambda}{x}}\left(1-\frac{x}{\mu}\right)\right),$$

$$\Phi_2(x)=\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(1+\frac{x}{\mu}\right)\right),$$

and $\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z}e^{-\frac{x^2}{2}}dx$ is the cdf of standard normal distribution.

Note: For $n=1$ the pdf in eq. (4) for the upper record value from inverse Gaussian distribution becomes parent inverse Gaussian distribution.

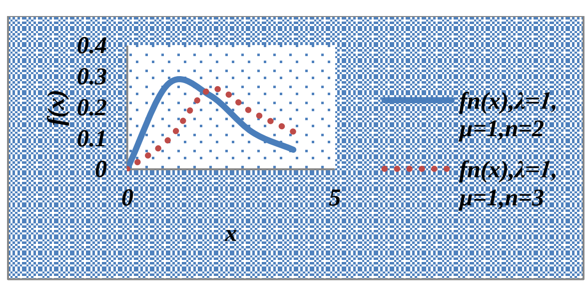

Fig. 1: pdf plot for $\lambda=1,\mu=1,n=2,3$

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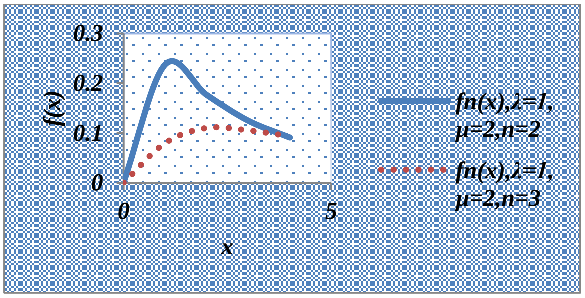

Fig. 2: pdf plot for $\lambda = 1, \mu = 2, n = 2,3$

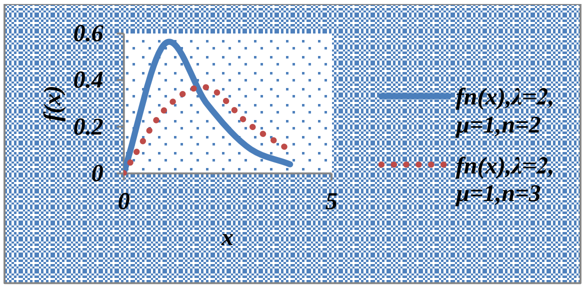

Fig. 3: pdf plot for $\lambda = 2, \mu = 1, n = 2,3$

Corollary **:** If X and Y are iid upper record functions from IGD. Then

a) $P(X < Y) = 1/2$,

b) $P(X < Y / x \geq m) = \left[(1 - F(m))^2 + F^2(m)\right]/2 \ \ , m > 0$,

c) $P(X < Y / x < y \leq m) = [1 - 2F(m)]/2 \ , m > 0$.

Limiting theorem of UR-IGD

In this section a limiting theorem UR-IGD is proposed as follows.

Theorem 1: Consider the pdf of upper record values from the inverse distribution in eq. (4) for $\mu = 1$ and $\lambda \to \infty$.Then the inverse Gaussian distribution upper record value distribution approaches to normal upper record value distribution with pdf

$$f_n(y) = \frac{1}{\sqrt{2\pi}\Gamma n} \exp\left(-y^2/2\right)\left[-\ln\left(1 - \Phi(y)\right)\right]^{n-1} \quad (5)$$

Proof:The pdf of upper record values from inverse Gaussian distribution for $\mu = 1$, in eq. (4) is

$$f_n(x) = \frac{\sqrt{\lambda}}{\sqrt{2\pi}\Gamma n} x^{-\frac{3}{2}} \exp\left(-\frac{\lambda(x-1)^2}{2x}\right)$$

$$\times\left[-\ln\left(\Phi\left(\sqrt{\lambda/x}(1-x)\right) - \exp(2\lambda)\Phi\left(-\sqrt{\lambda/x}(1+x)\right)\right)\right]^{n-1} dx$$

$$f_n(x) = \frac{\sqrt{\lambda}}{\sqrt{2\pi}\Gamma n} x^{-\frac{3}{2}} \exp\left(-\frac{\lambda(x-1)^2}{2x}\right)$$

$$\times\left[-\ln\left(\Phi\left(\sqrt{\lambda/x}(1-x)\right) - \exp(2\lambda)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left(-\frac{\lambda(1+x)^2}{2x}\right)dx\right)\right]^{n-1} dx. \quad (6)$$

Applying the transformation given below

$$\sqrt{\lambda}\frac{(x-1)}{\sqrt{x}} = y,$$

and

$$x = 1 + \frac{y^2}{2\lambda} + \sqrt{\left(1 + \frac{y^2}{2\lambda}\right)^2 - 1}$$

and taking $\lambda \to \infty$ in above expression eq.(6), and if $\lambda \to \infty \Rightarrow x \to 1$, then we get the pdf of upper record value Normal distribution

$$\underset{\lambda\to\infty}{Lim} f_n(y) = \underset{\lambda\to\infty}{Lim} \frac{1}{\sqrt{2\pi}\Gamma n} \exp\left(-y^2/2\right)$$

$$\times\left[-\ln\left(\Phi(-y) - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \exp\left(-\frac{\lambda}{2x}\left(1 + x^2 - 2x\right)\right)dx\right)\right]^{n-1} dy$$

$$f_n(y) = \frac{1}{\sqrt{2\pi}\Gamma n} e\left(-y^2/2\right)\left[-\ln\left(1 - \Phi(y)\right)\right]^{n-1}$$

The above equation is a pdf of upper record values from normal distribution.

Results and discussion

Some theoretical properties of UR-IGD

In this section we derive some theoretical properties of the upper record values from inverse Gaussian distribution (UR-IGD) such, mean, variance, median, and reliability measure as, survival function.

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Table 1 Mean of UR-IGD for $\mu = 1$

n \ λ	3	4	5	6	7	8	9	10
1	3.1	4.4	5.7	7.2	8.7	10.2	11.8	13.5
2	2.4	3.2	3.9	4.7	5.6	6.4	7.2	8.1
3	2.1	2.7	3.2	3.8	4.4	4.9	5.5	6.1
4	1.9	2.4	2.9	3.3	3.7	4.2	4.6	5.1
5	1.8	2.2	2.6	2.9	3.3	3.7	4.1	4.4
6	1.8	2.1	2.4	2.7	3.1	3.4	3.7	3.9
7	1.7	1.9	2.3	2.6	2.8	3.1	3.4	3.7
8	1.6	1.9	2.2	2.4	2.7	2.9	3.2	3.4
9	1.6	1.9	2.1	2.3	2.6	2.8	2.9	3.2
10	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

From the above table, for $\mu = 1$, as λ increase, mean of the upper record values from inverse Gaussian distribution decreases ($\lambda \to \infty$, mean $\downarrow$). But the mean of UR-IGD is increasing as n increasing.

Table 2 Variance of UR-IGD for $\mu = 1$

n \ λ	3	4	5	6	7	8	9	10
1	4.8	7.5	10.4	13.7	17.1	20.7	24.4	28.1
2	1.7	2.4	3.2	3.9	4.8	5.7	6.6	7.5
3	0.9	1.2	1.6	1.9	2.3	2.7	3.1	3.5
4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.1
5	0.4	0.6	0.7	0.8	0.9	1.1	1.2	1.4
6	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
7	0.3	0.3	0.4	0.5	0.5	0.6	0.7	0.8
8	0.2	0.3	0.3	0.4	0.4	0.5	0.5	0.6
9	0.2	0.2	0.3	0.3	0.4	0.4	0.4	0.5
10	0.17	0.20	0.23	0.27	0.30	0.33	0.37	0.40

From the above table, for $\mu = 1$, as λ increase, variance of the upper record values from inverse Gaussian distribution decreases ($\lambda \to \infty$, var.$\downarrow$). But the mean of UR-IGD is increasing as n increasing.

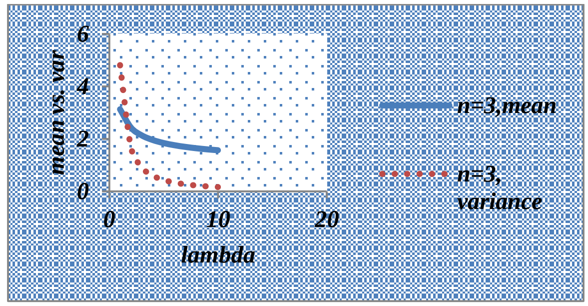

Fig. 4: mean vs. variance, $n = 3, \mu = 1, \lambda = 1, \ldots, 10$

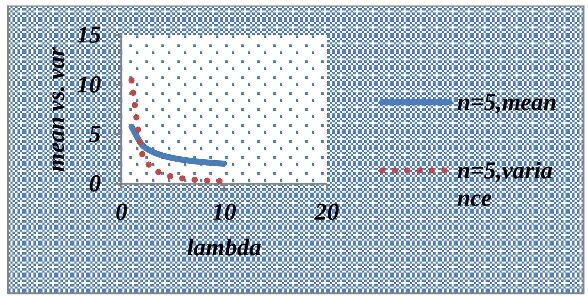

Fig. 5: mean vs. variance, $n = 5, \mu = 1, \lambda = 1, \ldots, 10$

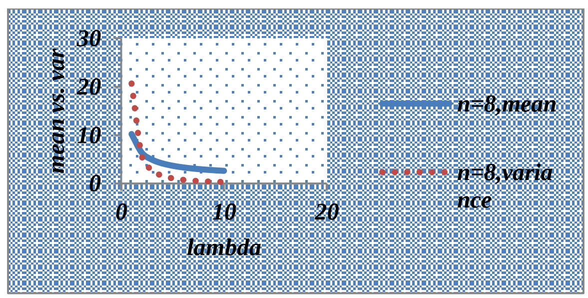

Fig. 6: mean vs. variance, $n = 8, \mu = 1, \lambda = 1, \ldots, 10$

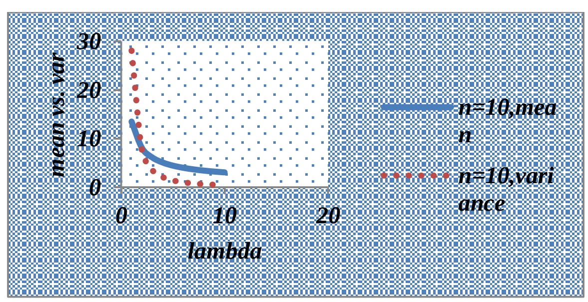

Fig. 7: mean vs. variance, $n = 10, \mu = 1, \lambda = 1, \ldots, 10$

The above figures 4-7, shows that mean and variance have same trend i-e in decreasing trend.

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Mean and variance both are in the same direction for λ increasing.

The Median of UR-IGD
The median of the UR-IGD is

$$\int_0^m f_n(x)dx = \frac{1}{2} \quad , \tag{7}$$

where $f_n(x)$ is the pdf of UR- IGD as in eq. (4).

Table 3 Median for $\mu = 1$

n \ λ	3	4	5	6	7	8	9	10
1	2.6	3.8	5.1	6.6	8.0	9.6	11.2	12.8
2	2.1	2.9	3.7	4.5	5.3	6.1	6.9	7.8
3	1.9	2.5	3.1	3.6	4.2	4.8	5.4	6.0
4	1.9	2.3	2.7	3.2	3.6	4.1	4.5	5.0
5	1.8	2.2	2.5	2.9	3.3	3.6	3.9	4.4
6	1.7	2.0	2.4	2.7	3.0	3.3	3.6	3.9
7	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.6
8	1.6	1.9	2.2	2.4	2.7	2.9	3.1	3.4
9	1.6	1.8	2.1	2.3	2.5	2.8	3.1	3.2
10	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0

From the above table, for $\mu = 1$, as λ increase, the median of the upper record values from inverse Gaussian distribution decreases. But the median of UR-IGD is increasing as n increasing.

Survival function of UR-IGD
Survival function is a property of any random variable usually associated with death or failure of some system, onto time. It is the probability that the system will survive beyond a specified time. The survival function of a continuous probability function $f(t)$ is by

$$S(t) = P(T > t) = \int_t^\infty f(t)dt$$
$$F(t) + S(t) = P(T \le t) + P(T > t) = 1.$$
$$S(t) = 1 - F(t).$$

The survival function of UR-IGD is

$$S_n(t) = \frac{\sqrt{\lambda/2\pi}}{\Gamma n} \int_t^\infty x^{-\frac{3}{2}} \exp\left(-\lambda(x-\mu)^2 / 2\mu^2 x\right) \times \left[-\ln\left\{\Phi\left(\sqrt{\frac{\lambda}{x}}\left(1-\frac{x}{\mu}\right)\right) - \exp(2\lambda/\mu)\Phi\left(-\sqrt{\frac{\lambda}{x}}\left(1+\frac{x}{\mu}\right)\right)\right\}\right]^{n-1} dx \tag{8}$$

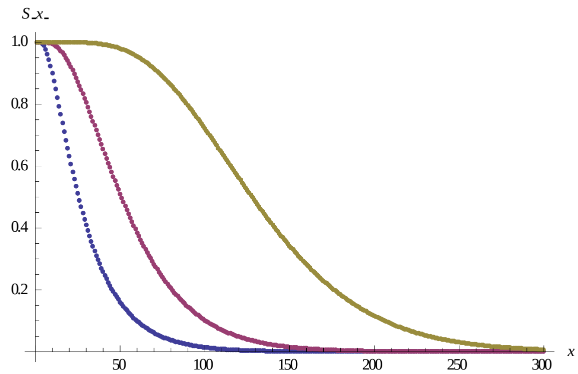

Fig. 8: Graph of $S(x)$ ***for*** $\lambda = 1, \mu = 1, n = 3,5,10$

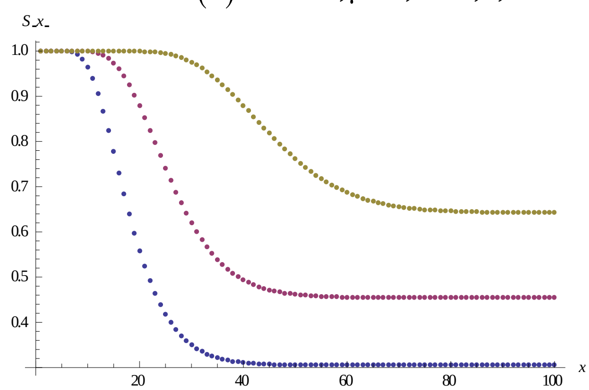

Fig. 9: Graph of $S(x)$ ***for*** $\lambda = 5, \mu = 1, n = 3,5,10$

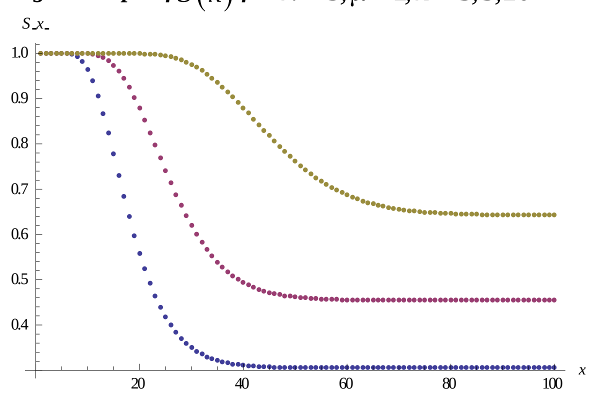

Fig. 10: Graph of $S(x)$ ***for*** $\lambda = 10, \mu = 1, n = 3,5,10$

From the above figures 8-10, it can be seen that the survival function of upper record values from IGD has decreasing trend. For large λ the survival

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function of upper record values from IGD is increasing. And the survival function of upper record values from IGD is increasing with n increasing.

Recurrence relation of UR-IGD

In this section a recurrence relation of single moments from UR-IGD has been established.

Theorem 2: For $n \geq 2$, the upper record values from the inverse Gaussian distribution, we have

$$\alpha_n^{(r)} = \frac{\lambda}{(2r+\lambda-1)}\alpha_n^{(r+2)} - \frac{2(\lambda-3)}{(2r+\lambda-1)\sqrt{\lambda}}\alpha_{n-1,n}^{(r+5/2,-1)} + \frac{2\sqrt{\lambda}}{(2r+\lambda-1)}\alpha_{n-1,n}^{(r+5/2)}, \qquad \text{for } n \geq 2. \tag{9}$$

Proof. For $n \geq 2$, we consider the rth moment of the upper record values from IGD is

$$\alpha_n^{(r)} = \frac{1}{\Gamma n}\int_0^\infty x^r\left[-\ln\left(1-F(x)\right)\right]^{n-1} f(x)dx$$

Since we have $f'(x) = \dfrac{df(x)}{dx}$, so $\dfrac{\lambda\left(1-x^2\right)-3}{2x} f(x)dx = df(x)$, integrating by parts, we get

$$\alpha_n^{(r)} = -\frac{1}{2(r+1)(n-1)!}\int_0^\infty x^r\left(\lambda\left(1-x^2\right)-3\right)\left[-\ln\left(1-F(x)\right)\right]^{n-1} f(x)dx$$

$$-\frac{(n-1)}{(r+1)(n-1)!}\int_0^\infty x^{r+1}\left[-\ln\left(1-F(x)\right)\right]^{n-2}\frac{f(x)}{1-F(x)}F'(x)dx$$

$$= \frac{(3-\lambda)}{2(r+1)}\alpha_n^{(r)} + \frac{\lambda}{2(r+1)}\alpha_n^{(r+2)}$$

$$-\frac{(n-1)}{(r+1)(n-1)!}\int_0^\infty x^{r+1}\left[-\ln\left(1-F(x)\right)\right]^{n-2}\frac{f(x)}{1-F(x)}F'(x)dx$$

$$= \frac{(3-\lambda)}{2(r+1)}\alpha_n^{(r)} + \frac{\lambda}{2(r+1)}\alpha_n^{(r+2)} - \frac{(n-1)}{\sqrt{\lambda}(r+1)(n-1)!}\times$$

$$\int_0^\infty x^{r+5/2}\left[-\ln\left(1-F(x)\right)\right]^{n-2}\frac{f(x)}{1-F(x)}f(x)dx. \tag{10}$$

Now using the property and hence

$$f(x) = \int_x^\infty f'(y)dy = \int_x^\infty \frac{\lambda\left(1-y^2\right)-3}{2y}f(y)dy,$$

So we can write eq. (1.8) as

$$\alpha_n^{(r)} = \frac{(3-\lambda)}{2(r+1)}\alpha_n^{(r)} + \frac{\lambda}{2(r+1)}\alpha_n^{(r+2)} - \frac{2(n-1)}{\sqrt{\lambda}(r+1)(n-1)!}$$

$$\int_0^\infty\int_x^\infty x^{r+\frac{5}{2}}\frac{\lambda\left(1-y^2\right)-3}{2y}\left[-\ln\left(1-F(x)\right)\right]^{n-2}\frac{f(x)}{1-F(x)}dydx$$

Simplifying above expression we get

$$\alpha_n^{(r)} = \frac{(3-\lambda)}{2(r+1)}\alpha_n^{(r)} + \frac{\lambda}{2(r+1)}\alpha_n^{(r+2)} - \frac{(\lambda-3)}{(r+1)\sqrt{\lambda}}\alpha_{n-1,n}^{\left(r+\frac{5}{2},-1\right)} - \frac{\sqrt{\lambda}}{(r+1)}\alpha_{n-1,n}^{(r+5/2)} \tag{11}$$

Rewriting the above equation we get the resulting equation (9).

Simulation study

In this section a simulation study is used to find mean, variance, median, mode and quartiles for upper record values from IGD. For this purpose, we take 50 upper record values of IGD in 50 random samples of 100 size each drawn from IGD. It is shown that for large λ the distribution of upper record values from IGD is symmetrical.

The following table shows the results of mean, variance, median, quartiles and mode of the upper record values from IGD for selected values of λ when $\mu = 1$.

Table 4 Simulations of Upper Record Values from IG Random Variables

λ	$E\left(X_{U(n)}\right)$	$\text{var}\left(X_{U(n)}\right)$	Median	Q_1	Q_3	Mode
1	6.11	4.38	5.21	4.56	8.16	5.18
3	3.24	0.40	3.05	2.77	3.62	2.94
10	2.0	0.08	2.0	1.86	2.23	2.0

From above table we have seen that for $\mu = 1, \lambda = 1$ the mean, median and mode are

Mean > Median > Mode,

where, Mean = 6.11, Median = 5.21, Mode = 5.18. The above relation shows that for $\mu = 1, \lambda = 1$, the distribution of upper record values from inverse Gaussian random variables is positively skewed. And for $\mu = 1, \lambda = 3$ the mean, median and mode are

Mean > Median > Mode,

where, Mean = 3.24, Median = 3.05, Mode = 2.94. The above relation shows that for $\mu = 1, \lambda = 3$, the distribution of upper record values from inverse Gaussian random variables is positively skewed. Hence for $\mu = 1, \lambda = 10$ the mean, median and mode are

Mean = Median = Mode,

where, Mean = Median = Mode = 2. The above relation shows that for $\mu = 1, \lambda = 10$, the distribution of upper record values from inverse Gaussian random variables is symmetrical. For $\mu = 1$ as $\lambda \to \infty$, the distribution of upper record values from inverse Gaussian random variables approaches to symmetry.

Conclusion

Record values have a pivotal role in many real life applications including data relating to sports, weather, economics, purchase order and any other type of documents. The inverse Gaussian distribution, a well known distribution provides an outstanding choice of probabilities model for a wide range of applications including reliability and life testing. In this paper, some theoretical properties including graphs of UR-IGD have been developed. The graphs of the pdf of UR-IGD shows that the distribution is right skewed. The mean, variance and median of UR-IGD have derived for different combinations of parameters. From these tables it has been concluded that for larger values of "lambda" mean, variances and median are decreases, but for larger values of "n" mean, variances and median are increases. Moreover for larger values of lambda mean and median approximately coincide. The graphs of the survival function shows that the survival function of UR-IGD is decreasing function. A limiting theorem has been established which shows that if the shape parameter lambda tends to infinity and scale parameter mu is one the distribution of upper values from inverse Gaussian approaches to the distribution of upper record values from normal distribution. The simulation study shows that for large values of lambda the UR-IGD becomes symmetrical. A lot of work has been done on record values from various distributions but still no literature has been found on record values from inverse Gaussian distribution. It is hoped that this paper will contribute usefull applications in record value theory.

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